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PRESSURE DROP IN TWO PHASE GAS–LIQUID FLOW IN INCLINED PIPES

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Abstract—Two phase pressure loss data are presented for angles from vertical upward to vertical downward for co-current air-water flow in a 4.55 cm diameter pipe operating at about atmospheric pressure. Detailed comparison with data from the literature indicated a substantial measure of agreement and showed that the method of data presentation suggested eliminates any effect on pressure loss of diameter, liquid density and system, but not that of liquid phase viscosity: Specific details are given on aspects of the measured frictional pressure loss from which it is deduced how energy losses can be minimised in two phase systems. In particular a total flow velocity of 15 ms^{-1} is recommended for most flow conditions in order to give a balance between flow capacity of the equipment and pressure loss.

INTRODUCTION

A large number of investigations have been carried out on the determination of pressure loss characteristics of two phase flow in horizontal and vertical conduits. Despite such work there exists a number of problems not least of which is the determination of the most suitable manner in which data should be presented. The variety of different techniques used by various workers to present data in this field indicates, among other things, that pressure loss in two phase flow can depend on a considerable number of variables. Indeed Ros (1961) used dimensional analysis to show that in two phase pressure loss 11 variables are involved leading to 7 dimensionless numbers which could be of significance in data presentation and formulation of theoretical or empirical predictions. There are a number of techniques, which are useful in single phase flow, which can be applied to two phase flow in order to narrow down the choice of variables. In general, the applications of these concepts, such as similarity, to the case of two phase flow seems to have fallen well short of expectation, particularly with certain flow regimes. The most rigorous analysis to date appears to be that developed successively by Hughmark (1963), Dukler *et al.* (1964) and Nguyen & Spedding (1973). This type of analysis indicates a similarity between the two phases inside the common conduit but its use is limited only to the case of symmetrical systems. Under these circumstances an approach to pressure loss prediction through modeling and semi-empirical correlation is perhaps more logical. Most models proposed to date usually are applicable, if not actually being derived for, a specific flow pattern or regime. Often use of these models has not been possible without a great deal of simplication or by extensive application of empirically derived factors. However, the results obtained can give reasonable predictions for a narrow range of application.

The most widely employed empirical correlation method appears to be of the Lockhart–Martinelli type (1949) where the use of two-phase friction multipliers, whose actual values have been determined empirically, have allowed helpful predictions to be made for certain flow regimes. Taitel & Dukler (1976) and Chen & Speeding (1981) have demonstrated that the method can be extended theoretically to give predictions for the separated flow regimes. Other important models were proposed by Hubbard (1965) for horizontal slug flow, while Griffith &

Wallis (1961) and Street & Tek (1965) provided noteworthy work on the vertical upward slug flow regime.

There has been little work reported in which two phase flow pressure loss is treated as a topic unique in itself. Notable among investigations which do not use the Moody chart or single-phase velocity profiles and related theories, are the experimental investigations of Govier *et al.* (1957, 1958, 1960, 1962) and the analysis of Solov'ev *et al.* (1967). Even here the investigation still are far from conclusive.

Comparatively little has been reported on a two phase flow pressure loss in inclined pipes. Table 1 sets out a summary of the investigations in this area.

Table 1. Inclined pressure loss investigations

Investigator	Pipe diameter cm	Angle to horizontal	Method	Particulars
Boeltner [1939]	1.905	+9.5°	D,E, air-water, air-oil.	$(\frac{\Delta P}{\Delta \ell})_{TP} \alpha \log (Q_G)$ at $\frac{W_G}{W_L}$ = constant.
Kosterin [1949]	various	+72°, +90°	D	f_{TP} / f_G vs. $\frac{Q_G}{Q_T}$ at Fr = constant.
Brigham [1957]	5.017	+55°, +12.4°	D, air-oil, air-water.	Data presented as $(\frac{\Delta P}{\Delta \ell})_{TP}$ vs G_G at G_L = constant.
Flanigan [1958]	10.16, 15.24, 20.32, 25.40, 40.64.	+ various	D,E, gas-oil.	$(\frac{\Delta P}{\Delta \ell})_f \alpha \bar{V}_{SG} (\frac{Q_G}{Q_L})^{0.32}$. Calculation method.
Sevigny [1962]	2.094	0°, +5°, +10°, +15°, +30°, +60°, +90°.	D,E	$f_{TP} \alpha \frac{Q_L}{Q_T}$, Re_L, Re_G. Holdup from input conditions.
Guzhov [1967]	5.08	0 to ±9	D,E	$f_{TP} / f_G \alpha \frac{Q_L}{Q_T}$ at Fr = constant.
Ney-Fuentes [1968]	1.27, 2.54	0°, +3°, +5°, +8°, +10°, +15°, +35°, +70°, +90°.	D	Data only.
Bonnecaze [1969, 1971]	various	0, ±2, ±6, ±10.	D,E,T	$f_{TP} \alpha Re_L$. Slug flow.
Singh [1970]	1.54, 2.088, 2.700, 3.475, 4.064.	0°, +1°, +5°, +10°, +15°, +17.5°.	D,T air-water.	$(\frac{\Delta P}{\Delta \ell})_f = 2f\rho_L \frac{V_T^2 \bar{R}_L}{D}$. Slug flow.
Nezhilskii [1970]	5.70	0°, ±1°, ±3°, ±5.5°, ±9.5°, ±14°.	D,E	$f_{TP} \alpha$ Fr, Re, $\frac{Q_G}{Q_T}$
Vermeuleu [1971]	1.27	0°, ±7°	D,T	$(\frac{\Delta P}{\Delta \ell})_f \alpha \bar{V}_T$, ρ_L, f. Slug flow.
Beggs [1972]	2.54, 3.81	0°, ±5°, ±10°, ±15°, ±20°, ±35°, ±55°, ±75°, ±90°.	D,E	$f_{TP} \alpha \bar{R}_L$, ℓ, flow properties.

T = Theoretical V_{SG} = Superficial gas velocity

E = Empirical

D = Data

Boeltner & Kepner (1939) presented inclined flow pressure drop data as logarithmic functions of the volumetric air rate Q_G at constant values of W_G/W_L gas to liquid mass ratios. These authors suggested that a general correlation might include Re, Reynold, Fr, Froude, and We, Webber numbers, but in fact no correlation was developed. The effect of flow rates and pipe diameter, D, on inclined pipe flow were studied by Kosterin (1949) and the ratio of two phase, f_{TP}, to single phase, f, friction factor was shown to be a function of input volumetric gas content and Froude number. General observations were made by Kosterin (1949) that the flow regime changed with pipe diameter and inclination, α, and that the effect of inclination was greatest at low liquid flow rates. Holdup data were not determined and therefore detailed calculations using the data are not possible. Brigham *et al.* (1957), Flanigan (1958) and Baker (1957) investigated and discussed pressure drop in two phase pipe lines over hilly terrain. Little contribution was made except to report that the total pressure drop for inclined upward flow always was greater than that in the corresponding case for horizontal flow. Again no attempt was made to measure holdup as such so the data are only of general value. Brigham *et al.* (1957) attempted to overcome the problem of holdup measurement by using a double loop system such that the pressure loss tapping points were at the same level and therefore did not require correction for static head. It was argued, that while the absolute value of the pressure loss may not be correct from such a geometrical arrangement of the apparatus, the relative values will be correct between the various angles of inclination tested and that of horizontal flow, since both systems will have the same geometrical path to traverse. Such an assumption may or may not be valid depending on whether any anomalous effects are induced due to the presence of bends in the flow path.

Guzhov *et al.* (1967) presented a correlation of a similar form to that developed by Kosterin (1949) for inclined pipes. Holdup also was correlated but their general friction factor correlation should be used with caution since the function they suggest becomes unbounded when the liquid flow tends to zero. Ney (1968) and Fuentes (1968) reported holdup and pressure loss data for upward flow in inclined pipes over an extremely narrow range of phase flow rates; so narrow in fact that the data only can be used to spot check an existing correlation and are not of wide application.

Singh & Griffith (1970) investigated slug flow in inclined pipes of different diameter and found an optimum pipe size existed, for constant flow rates of the fluid, at which the total pressure drop was a minimum. The same feature was apparent in the vertical upward flow data of Govier *et al.* (1957, 1958). The basic reason for the effect is that, for given flow conditions, the relative contribution to the total pressure drop due to elevation increases with pipe size while the frictional pressure drop contribution does the opposite. Singh & Griffith (1970) developed a simple model of slug flow by drawing on the work of others. This enabled the frictional pressure loss to be derived by neglecting the shear on the liquid film along the gas bubble and assuming that the frictional pressure loss depends only on the shear at the all liquid slug. Thus the frictional pressure loss is

$$[\mathrm{d}P/\mathrm{d}l]_f = 2 f_L \rho_L \bar{V}_T^2 \bar{R}_L D^{-1} \qquad [1]$$

where f_L was the Fanning friction factor for all liquid flow in smooth pipe, $\bar{V}_T$ was the total velocity, D the pipe diameter, and ρ_L the liquid density. Results obtained, together with data of Sevigny (1962), Bonderson (1969) and Parakh (1969), were claimed to fit the model within $\pm 15\%$. Later Bonnecaze *et al.* (1971) extended the general development by introducing a relation for liquid holdup, $\bar{R}_L$, and, f_{sL} a new friction factor obtained from a correlation of the total two phase pressure loss.

$$\left(\frac{\mathrm{d}P}{\mathrm{d}l}\right)_{TP} = \frac{\bar{R}_L - \bar{R}_{LF}}{1 - \bar{R}_{LF}} \left[g\rho_L \sin\alpha + \frac{2\rho_L f_{sF} \bar{V}_T^2}{D} \right] \qquad [2]$$

$$\bar{R}_{LF} = \frac{1}{\pi}(\cos^{-1}(1-n_s) - (1-\eta_s)\sqrt{(\eta_s(2-\eta_s))}) \qquad [3]$$

$$f_{sF} = 0.0048 + 3980/\mathrm{Re}_T^{1.285} \qquad [4]$$

where g is the gravitational acceleration, and η_s was the ratio of the liquid depth to pipe radius when the flow with the same $\bar{R}_L$ was assumed to be stratified. Unfortunately insufficient detail of data were supplied by both Singh & Griffith (1970) and Bonnecaze *et al.* (1969, 1971) to enable meaningful calculations to be performed.

Nezhilskii & Khodanovick (1970) measured pressure drop and holdup in a pipe inclined at various angles over a narrow range of flow rates. Empirical relations were developed for holdup and pressure loss. A pressure loss correlation was presented as a function of inlet gas content and the Reynolds and Froude numbers. Vermeulen & Ryan (1971) investigated slug flow of air and water in a pipe inclined at various angles. The flow was idealized with the assumption that the gas phase was incompressible and did not contribute to the pressure loss, while the no-slip condition pertained. Thus, if the slug frequency, ν, is known,

$$[\mathrm{d}P/\mathrm{d}l]_f = 2f\rho_L\bar{V}_T^2D^{-1}Q_LQ_T^{-1} + \rho_L\bar{R}_{LF}\bar{V}_T\nu \qquad [5]$$

where Q_L and Q_T are the liquid and total volumetric flow rates. Since the main consideration of the study was the degree of pressure fluctuation occurring in the two phase slug flow. Holdup data were not measured by Vermeulen & Ryan (1971); a circumstance which limits the usefulness of their data.

Nencetti *et al.* (1968a,b) investigated vertical downwards flow in the annular air-water system using 1.6, 2.0 and 2.4 cm diameter pipes. No correlation was attempted unlike Webb & Hewitt (1975) who reported detailed work on the 3.10 cm and 8.82 cm diameter tubes.

Beggs (1973) reported pressure drop and other two phase data for air-water flowing in inclined tubes of various diameters. The two phase friction factor was calculated from

$$f_{TP} = \frac{2D}{G_T\bar{V}_T}\left[\frac{\mathrm{d}P}{\mathrm{d}l}\left(1 - \frac{\rho_{TP}\bar{V}_T\bar{V}_{SG}}{P}\right) - (\rho_{TP}\,g\sin\alpha)\right] \qquad [6]$$

where $\rho_{TP} = \bar{R}_L\rho_L + \bar{R}_G\rho_G$, G_T is the total mass flow rate and $\bar{V}_{SG}$ is the superficial gas velocity. The friction factor was normalised with the no-slip friction factor, f_{NS}, calculated from the Moody chart for smooth pipe by using an empirical equation suggested by De Gance & Atherton (1970).

$$f_{NS} = [2\log[\mathrm{Re}_{NS}/[4.5222\log\mathrm{Re}_{NS} - 3.8215]]]^{-2} \qquad [7]$$

$$\mathrm{Re}_{NS} = \frac{(x_{qL}\rho_L + x_{qG}\rho_G)\bar{V}_TD}{\mu_Lx_{qL} + \mu_Gx_{qG}} = \frac{G_TD}{\mu_Lx_{qL} + \mu_Gx_{qG}} \qquad [8]$$

where μ is the viscosity.

The normalised friction factor f_{TP}/f_{NS} was then correlated against $x_{qL} = Q_L/Q_T$ with $\bar{R}_L$ as the other parameter. The method showed a fair accuracy when tested against the data from which the correlation was derived.

Because work on inclined pipes is less numerous than for horizontal and vertical conduits it is to be expected that any conclusions will be more indefinite for inclined pipes. Therefore the general comments already made about two phase pressure loss in horizontal and vertical flow definitely will apply to the case of inclined flow. It is the intention of this work to report experimental data on pressure loss for two phase flow in an inclined tube and to attempt to handle, in some measure, the associated data presentation and other problems.

EXPERIMENTAL

Two phase two component overall pressure loss is the sum of the losses due to friction, elevation and expansion. The latter is only of importance with $G_T \geq 2700\,\text{kg m}^{-2}\,\text{s}^{-1}$, and therefore can be ignored in the present study. The elevational pressure loss component is found from the corresponding holdup values which are given by Nguyen & Spedding (1977) who also have given the main details of the experimental apparatus. The pressure loss was measured over a 1.21 m test section, of 4.55 cm internal diameter perspex pipe, placed centrally between suitable calming lengths. The air and water mixture emerging from the apparatus was separated in a cyclone which was arranged so as to avoid back pressure waves being passed back to the test section. The tapping points were connected either directly to a piezometer ring sepatator or to a small cylindrical separation chamber in order to ensure either gas or liquid but not both were presented to the pressure tapping lines. The gauge pressure of the flow conduit and the pressure drop were measured by manometers. Generally the pressure fluctuated and therefore damping devices were necessary. If excessive fluctuations occurred, the pressure tapping lines from the lower side of the separators were used so that the pressure measuring lines were filled with water. These pressure measuring lines were connected to 18 litre, water filled, plastic containers which each then led to one leg of an overhead manometer. The two containers, one for each tapping point, were thin walled and therefore flexible enough to act as damping devices by accommodating any pressure fluctuations. The two overhead manometer legs were made of 1.27 cm glass tubing, in order to avoid miniscus effects, and were long enough to extend at least 1.5 m beyond the total height of the test rig when it was placed in the vertical position. The water level registered for the lower tapping point indicated the gauge pressure and since the lines were separated, there are minimal interaction due to any pressure fluctuations. The water level registered at the other top tapping point required correction for the different densities of the liquid and the two phase mixture. Thus the accuracy in measuring the frictional pressure drop in the non-horizontal mode depended on the accuracy of measurement of both the total pressure drop and the phase holdup. In one extreme where the liquid holdup as well as the change in elevation are high, the total pressure drop is made up mainly by the weight of the liquid content in the pipe line. Any scatter in the measured liquid holdup can easily cover up any trend which would otherwise be apparent in the frictional pressure drop. In this region of operation it was found to be essential to repeat the holdup and pressure loss measurement a number of times in order to achieve consistent frictional pressure loss data. When the pressure fluctuations were negligable the gas side pressure tapping line from the top of the sepators were used in order to provide better accuracy at low flow rates. For these readings an inclined methanol filled monometer was used. Temperature measurements were recorded and were consistently around room temperature but no attempt was made to regulate this variable.

The flow regime classification used in this work is that suggested by Spedding & Nguyen (1980).

RESULTS

It is desirable to consider certain theoretical aspects of the pressure loss in order to determine which is the best method of data presentation. Consider a cylindrical control volume of fluid as shown in figure 1. The force balance for the volume gives:

$$(P_1 - P_2)\delta A - \rho \delta l \delta A g \sin \alpha = \tau \qquad [9]$$

where A is the cylindrical area and τ is the total shear applied to the cylinder by the surroundings.

The subsequent analysis depends on the following two assumptions.

(i) The pressure drop between two points in the flow field, which have same co-ordinates with respect to the pipe wall, does not change with position across the pipe cross section.

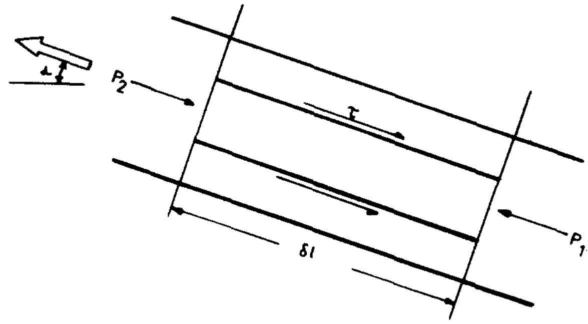

Figure 1. Cylindrical control volume.

(ii) There is always a liquid film which is continuous in the direction of mean flow on the pipe wall.

If the average of [9] is taken over a period of time it becomes:

$$(\bar{P}_1 - \bar{P}_2)\delta A - \bar{\rho}\delta l \delta A g \sin \alpha = \bar{\tau}. \qquad [10]$$

The quantity $(\bar{P}_1 - \bar{P}_2)$ is constant with respect to position according to assumption (i), but $\bar{\rho}$ and $\bar{\tau}$ are functions of position. Integration over the whole pipe cross-section is possible if information is provided on the functions which give the relationships between $\bar{\rho}$, $\bar{\tau}$ and position. The first term on the left hand side of [10] is $(\Delta P)_{TP}A$ which is the total pressure force difference between the two sections of the pipe. The term on the r.h.s. cannot be integrated as such but must equal the frictional force applied on the total fluid by the pipe wall $(\Delta P)_f A$. In addition it must be a function of τ_w. The remaining term $\bar{\rho}$ has been shown by Nguyen & Spedding (1977) to be

$$\bar{\rho} = \rho_L r_L + \bar{\rho}_G r_G \qquad [11]$$

where r is the point holdup or structural parameter, and is a function of position because r_L, r_G and $\bar{\rho}_G$ are dependent on position. The dependence on position of $\bar{\rho}_G$ is due to the existence of a profile of $\bar{P}$ and, in order to carry out the integration of this term, it is necessary to assume that $\bar{P}$ is constant wherever and whenever the gas phase is present in a cross sectional area segment. Thus [9] becomes the well known equation

$$\left(\frac{\mathrm{d}P}{\mathrm{d}l}\right)_{TP} - (\rho_L \bar{R}_L + \rho_G \bar{R}_G) g \sin \alpha = \left(\frac{\Delta P}{\Delta l}\right)_f. \qquad [12]$$

In seeking to choose dependent and independent variables of the system, with the presentation of data in view, table 1 shows that many workers attempted to use some type of two phase frictional factor associated in some manner to measureable quantities. It is obvious from the above development that there is an association between the pressure loss due to friction and some function of τ_w such that τ_w constitutes the independent variable. Hence, it is convenient to define a term called the shear or frictional velocity using liquid properties,

$$U^* = \sqrt{(\tau_w/\rho_L)} \qquad [13]$$

where τ_w is the time-average shear force at the wetted wall of the pipe. However from assumption (i) the shear force in the axial direction is independent of position and is in fact a measure of the pressure difference caused by friction at the pipe wall. Thus,

$$\tau_w \, \mathrm{d}l \pi D = (\mathrm{d}P_f \pi D^2)/4 \qquad [14]$$

which gives from [13],

$$U^* = \sqrt{\left(\left(\frac{\Delta P}{dl}\right)_f \frac{D}{4\rho_L}\right)} \qquad [15]$$

U^* is used here as the dependent variable and is calculated from [12] and [15]. The independent variables are the flow rates of the two phases but there are a number of forms that they can take. A detailed inspection of actual data showed that the use of $\bar{V}_T$ as the independent variable gave an acceptable presentation of the data with the superficial liquid velocity $\bar{V}_{SL}$ being involved as a minor variable. The data are presented as plots of U^* against $\bar{V}_T$ for various angles of inclination in figures 2–12 or in tabular form (Spedding & Nguyen 1976). The precision of the data varied with flow regime but can be ascertained from the detailed tabular results.

DISCUSSION

The data are worthy of initial discussion in a general way as they exhibit certain characteristics feactures which are dependent on flow regime. The general picture of the relation between frictional velocity or pressure loss against total velocity can be gained by examination of figure 7 containing the data for horizontal flow. This form of the relationship of pressure loss and increasing flow rate is in general agreement with that observed by Eaton (1966). In the main a family of curves are formed which exhibit a systematic increase in frictional pressure loss as the superficial liquid velocity is increased. Initially at low values of Q_L and $\bar{V}_T$ the frictional velocity is in the stratified regime and is independent of $\bar{V}_T$ giving no other than the free-surface channel flow value which is a function of $\bar{V}_{SL}$ alone. U^* possesses an increasing relation with $\bar{V}_T$ for the condition that $\bar{V}_T > 1.0\,\mathrm{m\,s^{-1}}$. The flow patterns covered in this increasing region of the relation are the stratified wavy, the annular, the annular plus droplet and the film regimes. As the liquid velocity is increased the bubble, slug and annular blow-through slug regimes appear when $\bar{V}_T < 10.0$ m/s. These regimes are characterised by

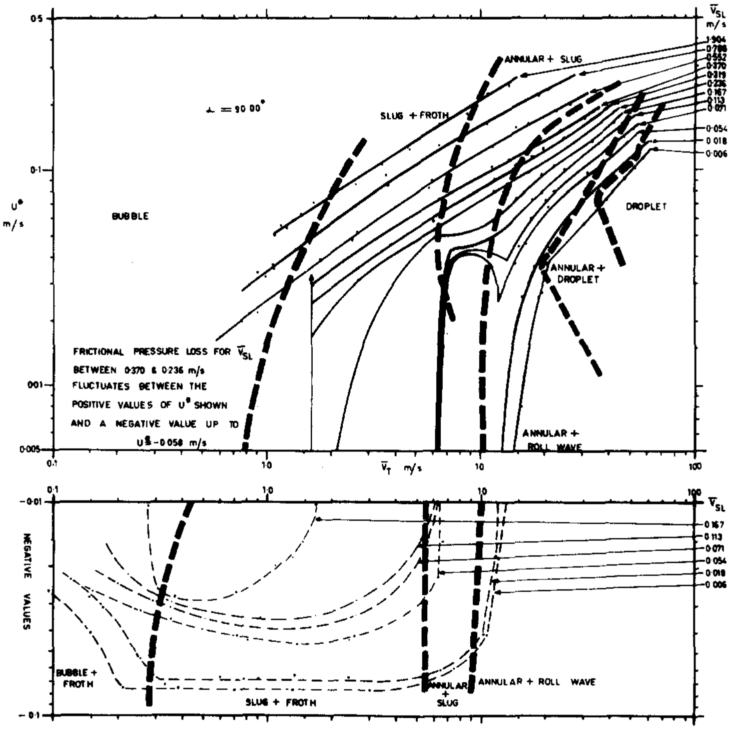

Figure 2. Frictional velocity against total velocity for air–water two phase flow in a 4.55 cm dia. tube in the 90° vertical mode. The negative value of U^* arises because the frictional pressure loss attains negative values for certain low gas rate regimes. These regimes are shown as dashed lines. For the $\bar{V}_{SL}$ velocities between 0.236 to 0.370 m s^{-1} the frictional pressure loss fluctuates between positive and negative values when the total velocity was $\bar{V}_T < 1.6\,\mathrm{m\,s^{-1}}$ as shown by the vertical arrow on the diagram.

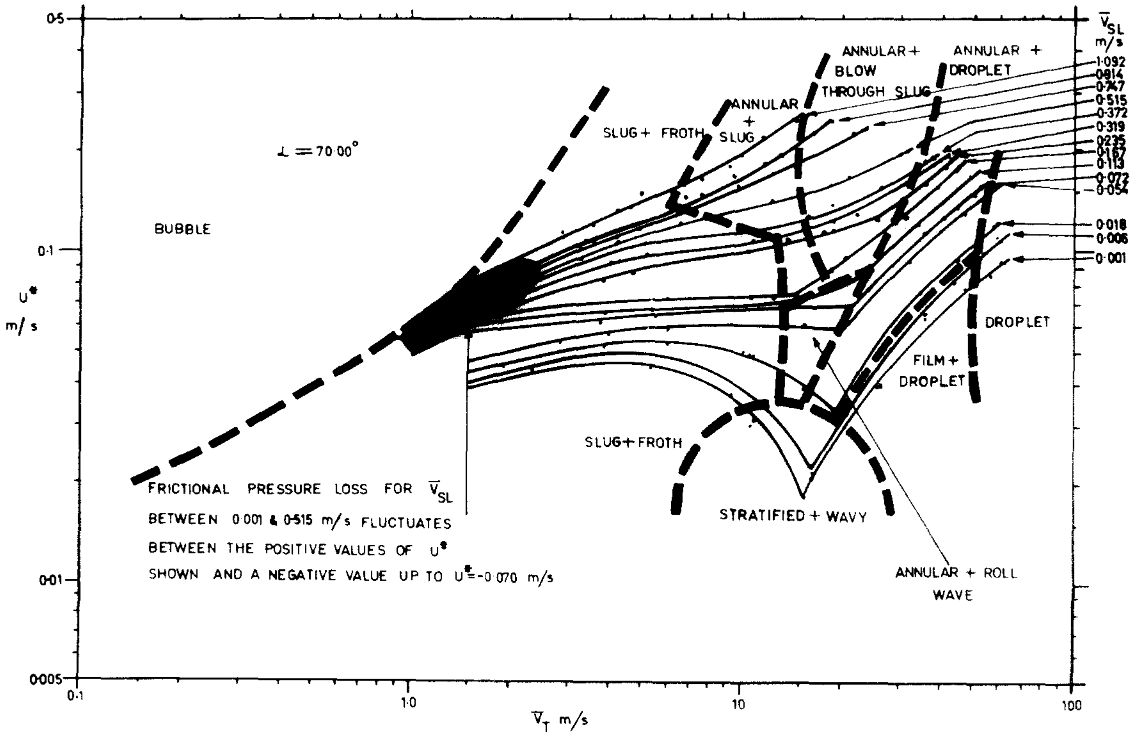

Figure 3. Frictional velocity against total velocity for air–water two phase flow in a 4.55 cm dia. tube in the 70° upward mode. For the $\bar{V}_{SL}$ velocities between 0.001 to 0.515 m s^{-1} the frictional pressure loss fluctuates between positive and negative values when the total velocity was $\bar{V}_T < 1.5$ m s^{-1} as shown by the vertical arrow on the diagram.

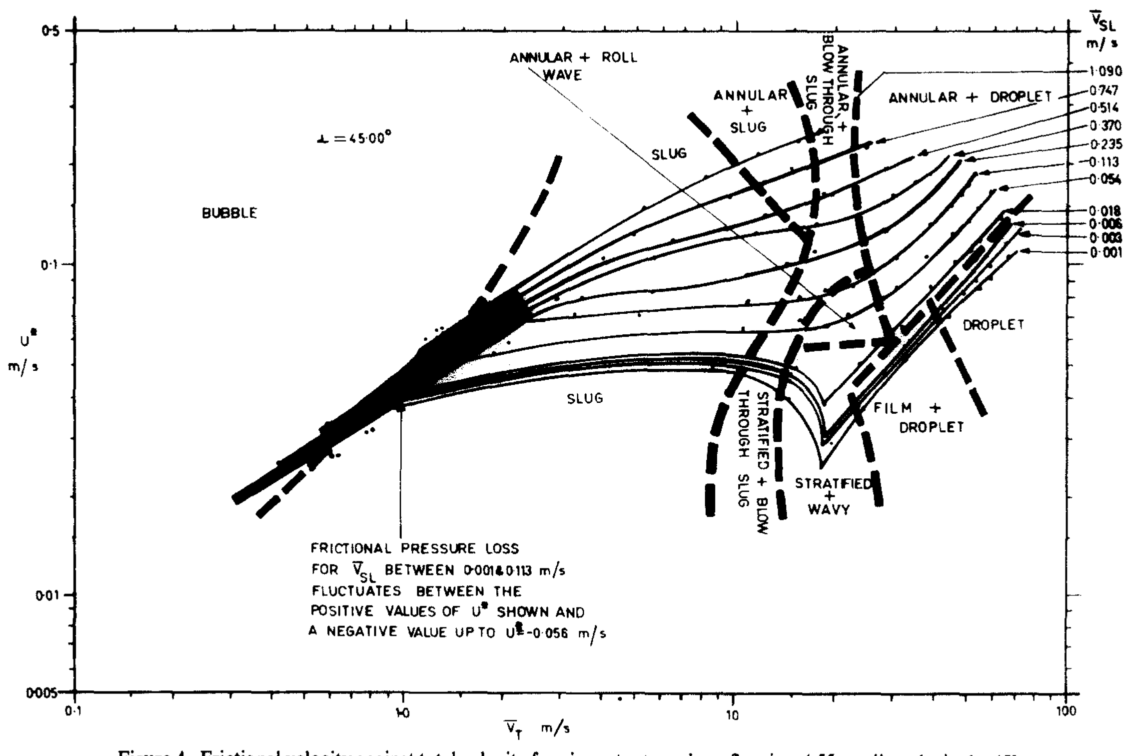

Figure 4. Frictional velocity against total velocity for air–water two phase flow in a 4.55 cm dia. tube in the 45° upward mode. For the $\bar{V}_{SL}$ velocities between 0.001 to 0.113 m s^{-1} the frictional pressure loss fluctuates between positive and negative values when the total velocity was $\bar{V}_T < 1.0$ m s^{-1} as shown by the vertical arrow on the diagram.

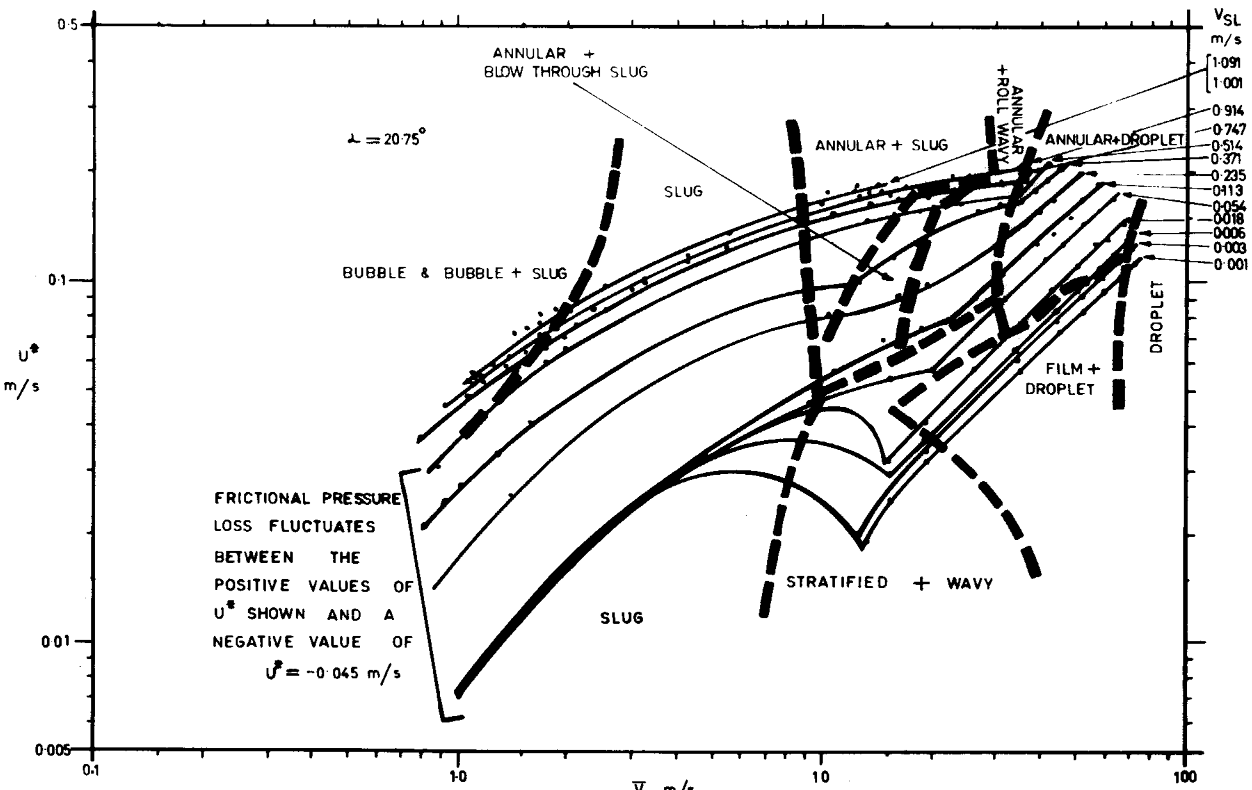

Figure 5. Frictional velocity against total velocity for air–water two phase flow in a 4.55 cm dia. tube in the 20.75° upward mode. For the $\bar{V}_{SL}$ velocities between 0.001 to 0.514 m s^{-1} the frictional pressure loss fluctuates between positive and negative values when the total velocity was approximately $\bar{V}_T < 1.0$ m s^{-1} as shown in the bracket on the diagram.

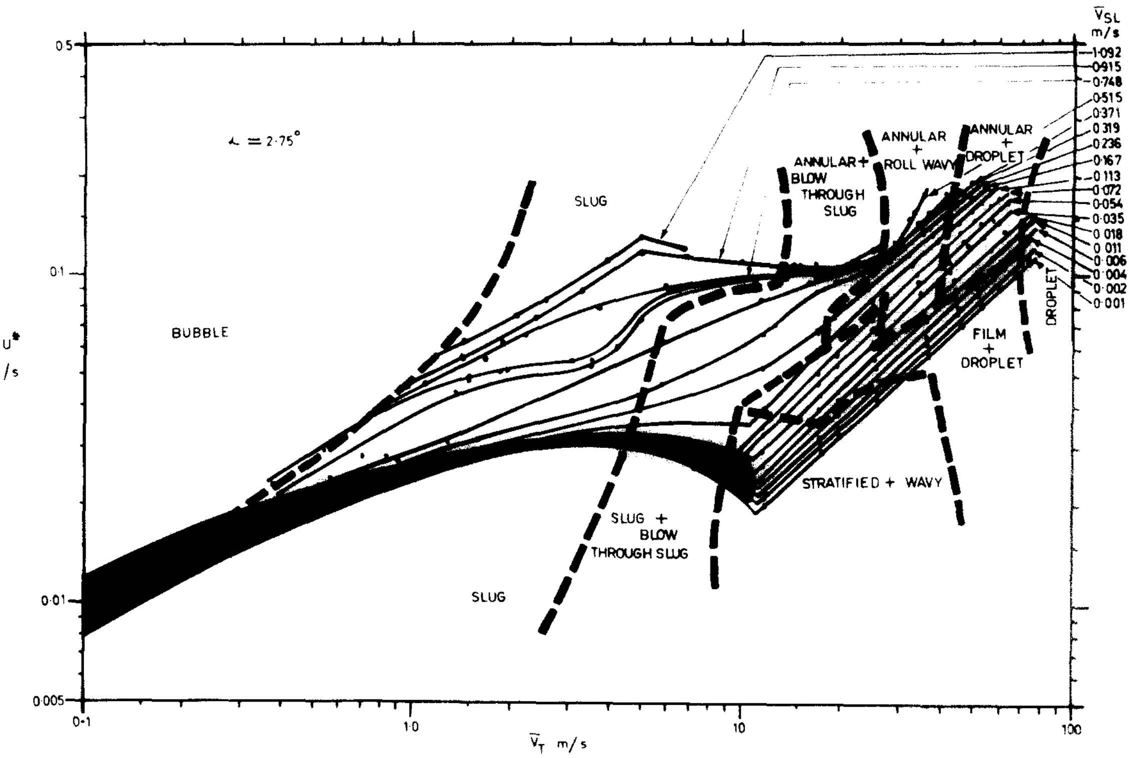

Figure 6. Frictional velocity against total velocity for air-water two phase flow in a 4.55 cm dia. tube in the 2.75° upward mode.

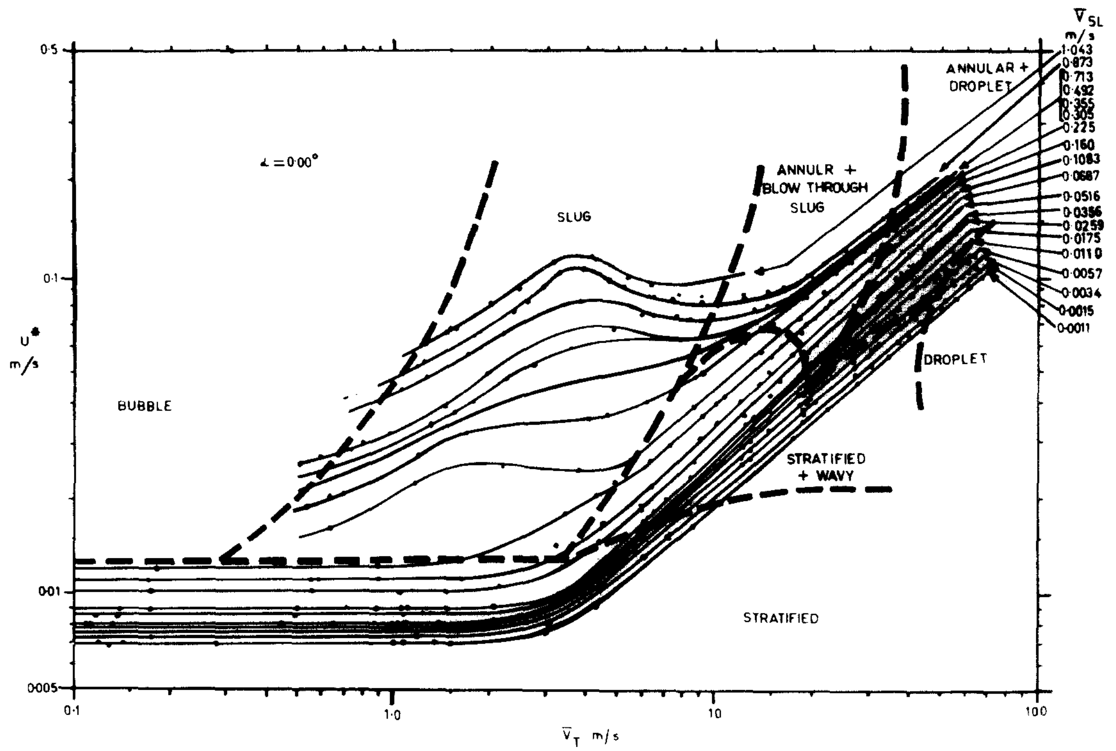

Figure 7. Frictional velocity against total velocity for air-water two phase flow in a 4.55 cm dia. tube in the horizontal mode.

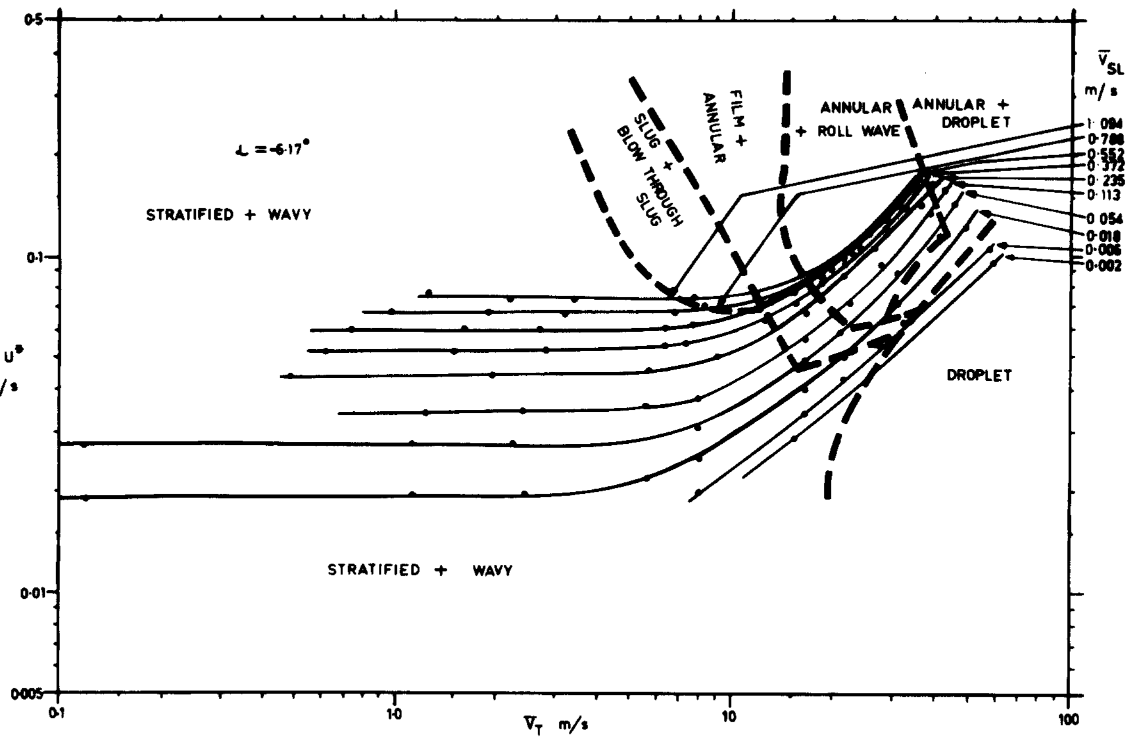

Figure 8. Frictional velocity against total velocity for air–water two phase flow in a 4.55 cm dia. tube in the 6.17° downward mode.

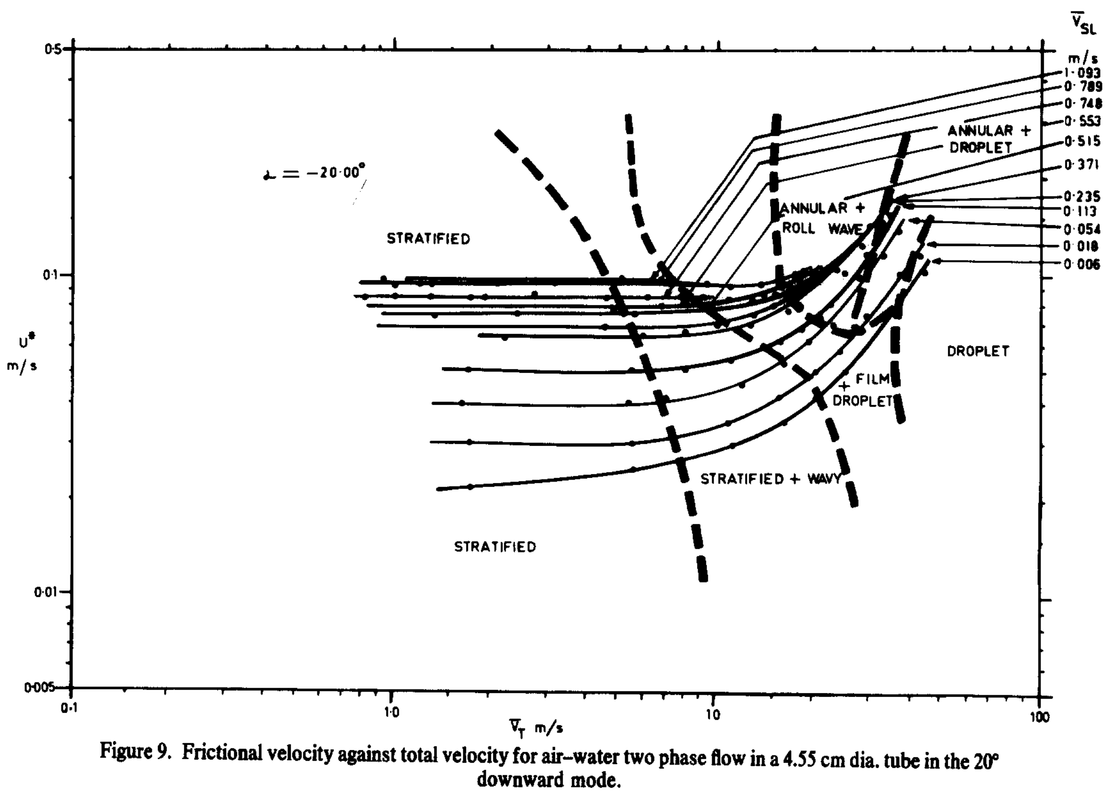

Figure 9. Frictional velocity against total velocity for air–water two phase flow in a 4.55 cm dia. tube in the 20° downward mode.

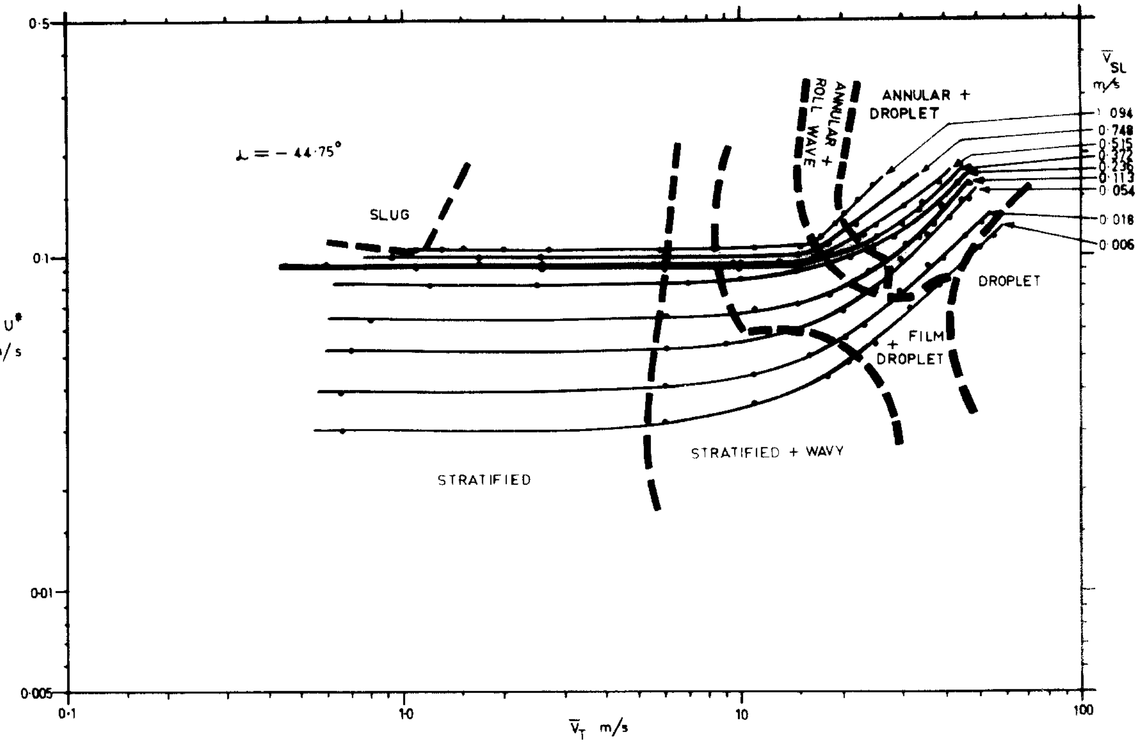

Figure 10. Frictional velocity against total velocity for air–water two phase flow in a 4.55 cm dia. tube in the 44.75° downward mode.

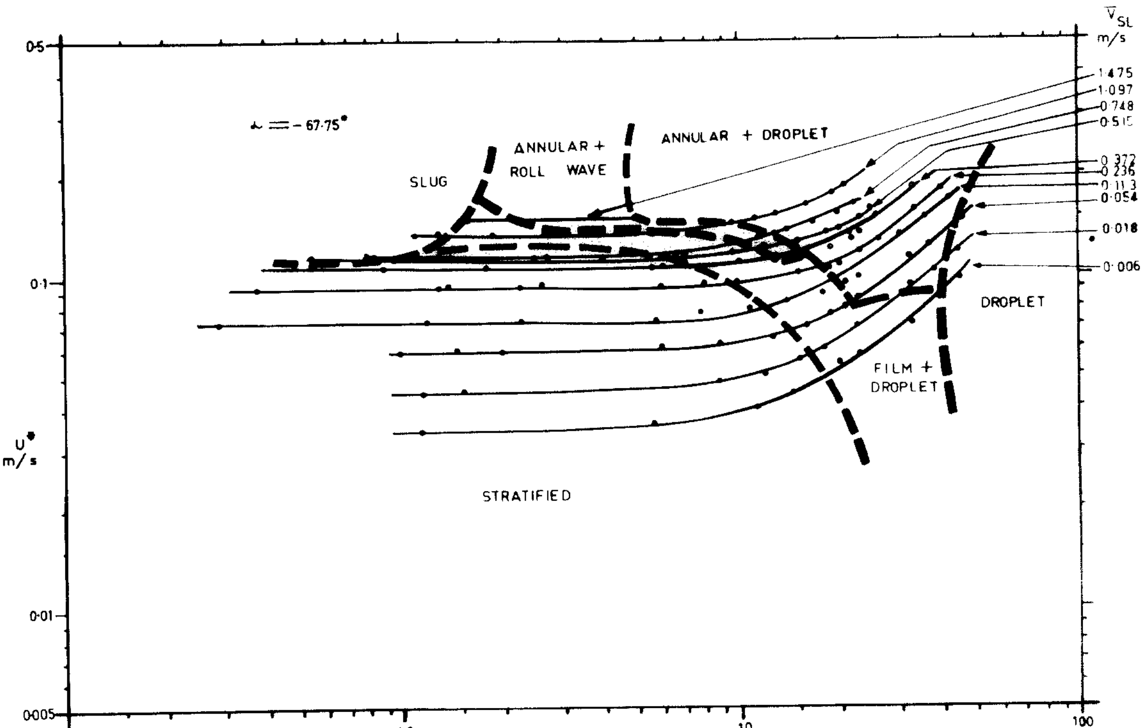

Figure 11. Frictional velocity against total velocity for air–water two phase flow in a 4.55 cm dia. tube in the 67.75° downward mode.

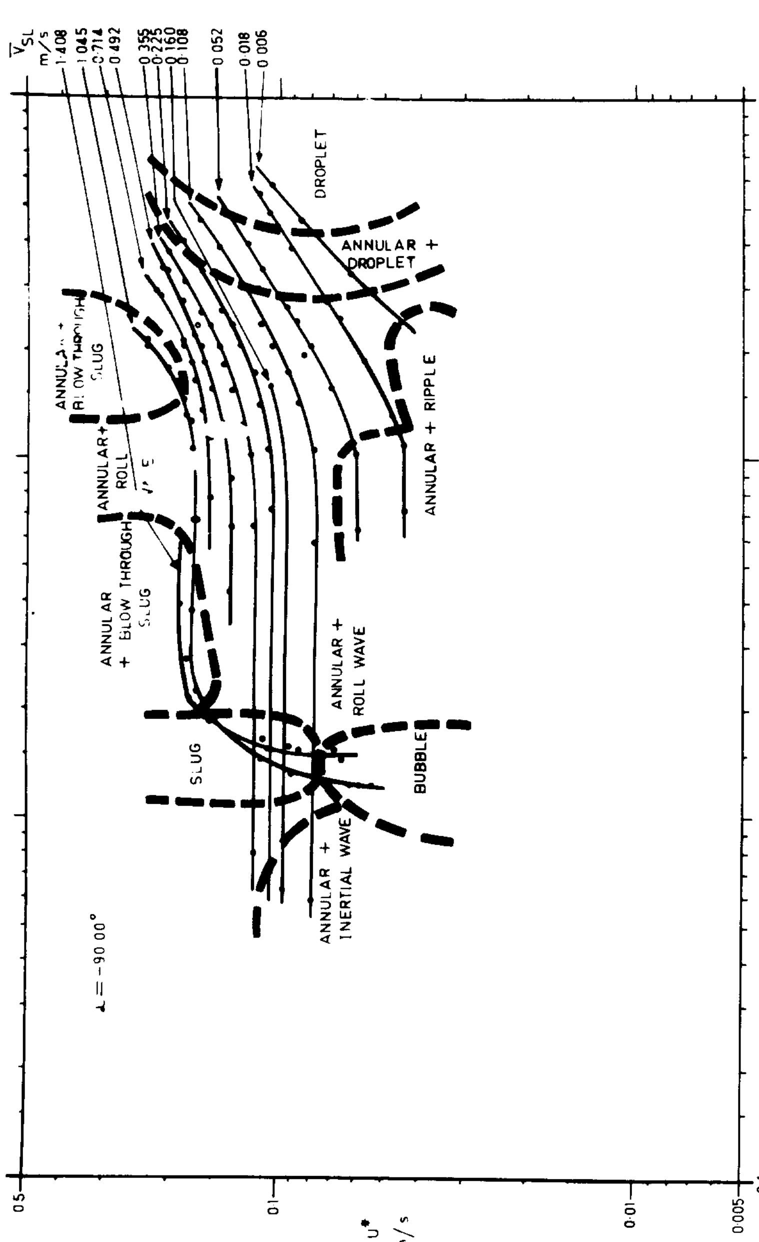

Figure 12. Frictional velocity against total velocity for air–water two phase flow in a 4.55 cm dia. tube in the 90° downward mode.

correspondingly much larger fluctuating pressure losses with a maximum in the U^* value appearing in the slug regime at $\bar{V}_T$ values of between 1.5 and 3.5 m s^{-1}. When positive angles of inclination are given to the pipe flow the stratified regime corresponding to free-surface channel flow is eliminated and, in the main, is replaced by slug flow which is characterised by higher fluctuating pressure loss values with an associated maximum. The rising portion of the U^* vs $\bar{V}_T$ relationship at $\bar{V}_T > 1.0$ m s^{-1} is maintained for a similar progression of flow regimes as was observed for horizontal flow. More importantly as the angle of inclination is increased the corresponding frictional pressure loss is increased, in general, except for a narrow region around $\bar{V}_T \approx 15$ m s^{-1} where the pressure loss remains consistently of the same order as in horizontal flow except for the case of near vertical flow.

By contrast when a negative angle of inclination is given to the pipe the stratified plus wavy regime is extended to cover the region $\bar{V}_T < 20$ m s^{-1} until, at the vertical downward flow position, the annular type regime is achieved. The maximum in the pressure loss associated with the slug regime vanishes. However, it should be understood that may be only because the liquid flow rate necessary for such a flow regime to exist cannot be reached in a sufficiently wide range under the conditions of these experiments. In general downward flow is characterised by frictional pressure losses which are higher than those of the corresponding horizontal case.

For horizontal flow, the initial increase in frictional velocity in the bubble regime may be attributed to the energy losses due to vigorous gas–liquid interactions as the bubbly mixture moves almost homogeneously down the conduit. As the gas velocity is increased the relative volume of the gas in the pipe rises causing the bubbles to agglomerate to form the characteristic slug flow regime. The pressure loss initially rises as the slug flow regime is developed due to the existence of well developed circulation patterns in the leading portion of the liquid slugs. Goldsmith & Mason (1962) have shown that, in the slug flow regime, the liquid film surrounding the moving gas bubble remains stagnant. Thus those portions of the inside surface of the tube which are occupied by the liquid slug contribute towards the pressure loss while those adjacent to the gas slugs impart very little to the frictional loss. Therefore as the gas velocity is increased in the slug regime the relative proportion of gas bubbles in the conduit increases leading to a maximum in the frictional velocity as the overall energy losses are reduced. In the blow-through slug and the annular type flow regimes there is a rapid build up of pressure loss indicating the presence of a large energy consuming interaction between the gas and liquid phases. This region of the data illustrates the general feature that a change in slope of the curves usually corresponds to a change in the type of flow pattern.

For vertical downwards flow the basic physical reason for the high frictional velocities recorded is that the bouyancy forces on the gas elements oppose the general downward motion of the two phase mixture. The observations in this present work for downward flow are in agreement with the findings of Bonnecaze *et al.* (1969, 1971) who observed that it is possible for slug velocity to be almost stationary because of the opposing effect of the bouyancy forces. Vertical upward flow has an opposite effect in that the bouyancy forces aid the general motion of the two phase mixture. However as discussed by Spedding & Nguyen (1978) the negative frictional effects directly attributable to bouyancy forces in upward inclined flow only become important in the case of vertical or near vertical flow.

A number of points are clear from the discussion of the data so far. Firstly the frictional velocity is dependent on the flow regime present in the pipe. Secondly, the slug flow regime for horizontal and inclined upward flow particularly should be avoided since it is characterised by high pressure losses of a fluctuating nature. Minimal frictional velocities for horizontal flow can be achieved by operating at a total flow velocity around 15 m s^{-1} in the stratified or stratified wavy regimes. For vertical or near vertical flow negative frictional pressure losses are recorded for the conditions $\bar{V}_T < 18$ m s^{-1} and $\bar{V}_{SL} < 0.1$ m s^{-1}. Finally as a general rule the frictional velocity of downward flow is higher than the corresponding horizontal case.

A detailed examination was conducted of the data, empirical correlations and theoretical models for inclined flow which are available in the literature to ascertain if agreement exists with the present work. It was found that detailed comparison was not possible in a number cases because essential data such as the holdup, were lacking. (Kosterin 1949; Brigham *et al.* 1957; Flanigan 1958; Servigny 1962; Bonderson 1969; Parakh 1969; Singh & Griffith 1970; Vermeulen & Ryan 1971). The data by Boeltner & Kepner (1939) were in the medium liquid flow, low total flow regime of $\bar{V}_T < 0.17\ \mathrm{m\ s^{-1}}$ where fluctuating conditions can be expected. Therefore the data are of limited usefulness but in general appeared to give a higher result than in the present work. General agreement was obtained with the data of Guzhev *et al.* (1967) and with the limited data of Ney (1968), Fuentes (1968) and Nezhilskii & Khodanovich (1970). The more extensive data of Beggs (1973). showed close agreement with the present work except at the lower liquid flow region of $\bar{V}_{SL} < 0.045\ \mathrm{m\ s^{-1}}$ where pressure losses were greater than in the present work. The divergence at low liquid flows may be due to certain geometrical aspects of Beggs' apparatus which resulted in obtaining insufficient calming length in the low liquid flow region. For vertical downward flow the limited data of Nencetti *et al.* (1968 a,b) and the extensive data of Webb & Hewitt (1975) showed good agreement with that of the present work. Almost all of the empirical correlations failed to give any close agreement with the data of the present work (Flanigan 1958; Guzhov *et al.* 1965, 1966, 1967; Singh & Griffith 1970; Servigny 1962; Beggs 1973). The best of the correlations tested proved to be that of Nezhilskii & Khodanovich (1970) and even in that case the agreement was not always satisfactory particularly in the bubble and slug flow regimes.

The theoretical models proposed by Singh & Griffith (1970), Bonnecaze *et al.* (1969, 1971) and Vermeulen & Ryan (1971) for the slug flow regime all gave unsatisfactory predictions when compared with the data of this work. The comparisons are perhaps rather unfair in light of the fluctuating nature of the results particularly in the lower total flow regime of $\bar{V}_T < 1.0\ \mathrm{m\ s^{-1}}$ as evidenced by figures 3–5. However data reported by other workers for the vertical and horizontal cases also failed to agree with the predictions of these models. It should be understood that when the term agreement is used in the above comparisons the respective data are the same within the normal error spread expected for the experimental results of this work. Data outside of this range are deemed to be unsatisfactory.

Much data has been presented in the literature for the cases of vertical and horizontal flow, and since a great deal of the published data on inclined flow did not support or could not be used to support that of this work, it was deemed prudent to ascertain if published results for horizontal vertical flow supported the work presented here. Data by Reid *et al.* (1957), James & Silberman (1958), Wicks & Dukler (1960) and Beggs (1972) for horizontal two phase air–water flow showed good agreement with this work despite being obtained from a wide range of pipe diameters (Spedding & Chen 1980a). It is well known that both pipe diameter and liquid mass velocity have an effect on the frictional pressure loss obtained experimentally. For example with the prediction of pressure loss using the Lockhart—Martinelli type of approach, experimental data in the low liquid mass flow and small diameter region given higher values than predicted. As the liquid mass flow and diameter are increased then the experimental data approaches the predicted values. Indeed, Speeding & Chen (1980b) have extended the Lockhart–Martinelli approach to enable the pressure loss to be predicted for the case of separated flow. In addition these authors detail how the predicted frictional pressure loss and the experimental diverge as the pipe diameter and liquid flow rate are decreased for the annular flow regime. Because the method of data presentation suggested in this work successful represented literature data for a wide range of pipe diameters the technique can be used with confidence over a wide range of geometries. A word of caution requires to be struck regarding the high superficial liquid velocity region of $\bar{V}_{SL} = 1.5\text{–}7.7\ \mathrm{ms^{-1}}$. Data from the literature indicates that at these high liquid flows the bubble regime extends up to $\bar{V}_T = 10\text{–}20\ \mathrm{ms^{-1}}$ so that no slug regime is encountered at all as the gas flow is increased. This means that, for example, in figure 7 for horizontal flow, the slug regime as shown does not extend much above $U^* = 0.2 \text{ m s}^{-1}$ since data in this area would be in the bubble regime and would not exhibit any pressure loss maximum.

Other data from the literature for horizontal flow exhibited agreement with this work in some areas of operation only. For example the data of Baker (1954), Andrews (1960), Govier & Omer (1962) and Eaton (1966) all give good agreement at higher liquid flows but at low liquid flows gave higher pressure loss values than the corresponding results of this work. By contrast the data of Chisholm & Laird (1958) gave agreement only at low liquid flows. Johnson & Abou-Sabe (1952) reported results which were in general beyond the liquid flow range used in this work but where comparison could be made their data gave higher values of frictional pressure drop than those reported here although some agreement was registered at low gas flow rates. The steam–water data of Harrison (1975) gave some agreement with the results of this work but about 60% of their data gave readings which were on the higher side. The reason for these high readings is unclear particularly since good agreement was obtained with the vertical flow data. Johnson (1955), McMillan *et al.* (1964) and Eaton (1966) all showed that the effect of an increase in liquid viscosity was to increase the frictional pressure loss recorded over that given by the air–water system. Increases in frictional pressure loss of 60% to 120% (depending on gas rate) were typical if the liquid viscosity was increased to $2.5 \times 10^{-2} \text{ kg m}^{-1} \text{ s}^{-1}$.

For vertical two phase flow the air–water data of Brown *et al.* (1960) Hewitt *et al.* (1961) and Beggs (1972), the steam–water data of Isbin *et al.* (1957) and Harrison (1975), and the potassium metal data of Alad'yev *et al.* (1964) showed good agreement with this work. Other data by Moore & Wilde (1931) and Govier & Short (1958) gave high results for a little under half of their reported data in a manner which showed no consistent trend. In addition the work of Moore & Wilde (1931) confirmed that the effect of increasing liquid viscosity was to increase the registered pressure loss. It is of interest to note that the data for steam-water and potassium are successfully predicted by figure 2 despite the fact that the liquid density of potassium is significantly different to that of water for which the graph in this work was determined. Again this highlights the success of the method of data presentation suggested in this work. It can be deduced from the complexity of the frictional pressure loss data presented here that it will be extremely difficult to devise a general purpose correlation or theory which will enable reliable prediction to be made. A more practical solution appears to be to devise a prediction method for each of the major flow regimes encountered in two phase systems.

There are a number of distinctive features of the pressure loss data which show up more effectively if the data are plotted against angle of inclination as shown in figures 13–20. The first four of these graphs show the total pressure loss data which are of considerably more practical interest than the frictional pressure loss data. At lower superficial liquid velocities were $\bar{V}_{SL} < 0.4 \text{ ms}^{-1}$ a definite pattern is apparent in the total pressure loss results with a distinct change in the characteristic somewhere between $\bar{V}_T = 10$ to 30 ms^{-1} where the annular type flow regimes commence to be observed. For the range of $\bar{V}_T < 10 \text{ ms}^{-1}$ there is a maximum in the total pressure loss at an angle of between 60° and 70°, falling rapidly away to a low value which is reached at the horizontal condition. Thereafter, for downward flows the total pressure loss is very low with a tendency to give slightly negative values for the case of vertical downward flow at the lowest total liquid velocity. By contrast, for upward vertical flow the highest total pressure losses are recorded for the lowest total flow rate while a minimum pressure loss is achieved at a total flow velocity of about 15 ms^{-1} where the total pressure loss is almost idependent of angle of inclination. At lower superficial liquid velocities, i.e. $\bar{V}_{SL} < 0.4 \text{ ms}^{-1}$ and total velocities $\bar{V}_T > 30 \text{ ms}^{-1}$, the upward flow conditions is at first almost independent of angle at a low value of total pressure loss. As the total velocity is increased a maximum appears in the total pressure loss at an angle of approximately 45°. A decided minimum is apparent at the horizontal condition so that any slight variation in angle either to the positive or negative slope results in a substantial increase in total pressure loss. For

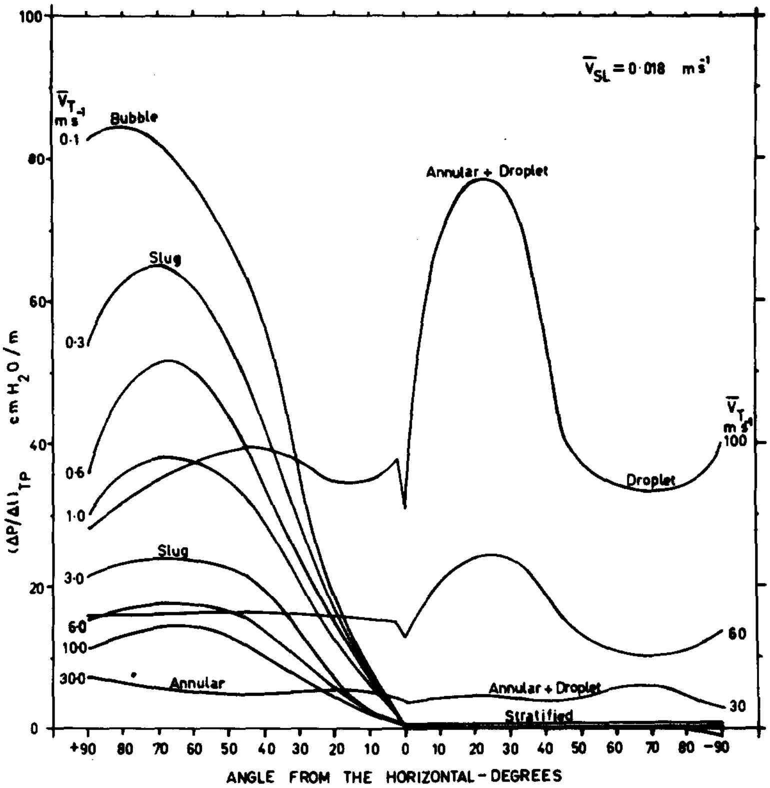

Figure 13. Two phase total pressure loss against angle of inclination for the air–water system in a 4.55 cm dia. tube at a superficial liquid velocity of 0.018 m s^{-1}.

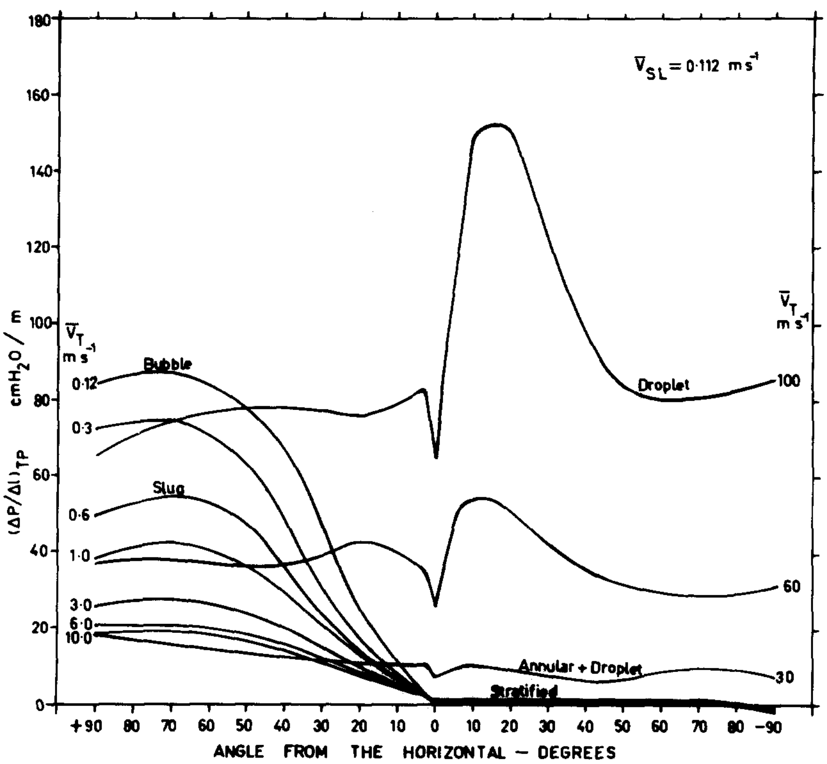

Figure 14. Two phase total pressure loss against angle of inclination for the air–water system in a 4.55 cm dia. tube at a superficial liquid velocity of 0.112 m s^{-1}

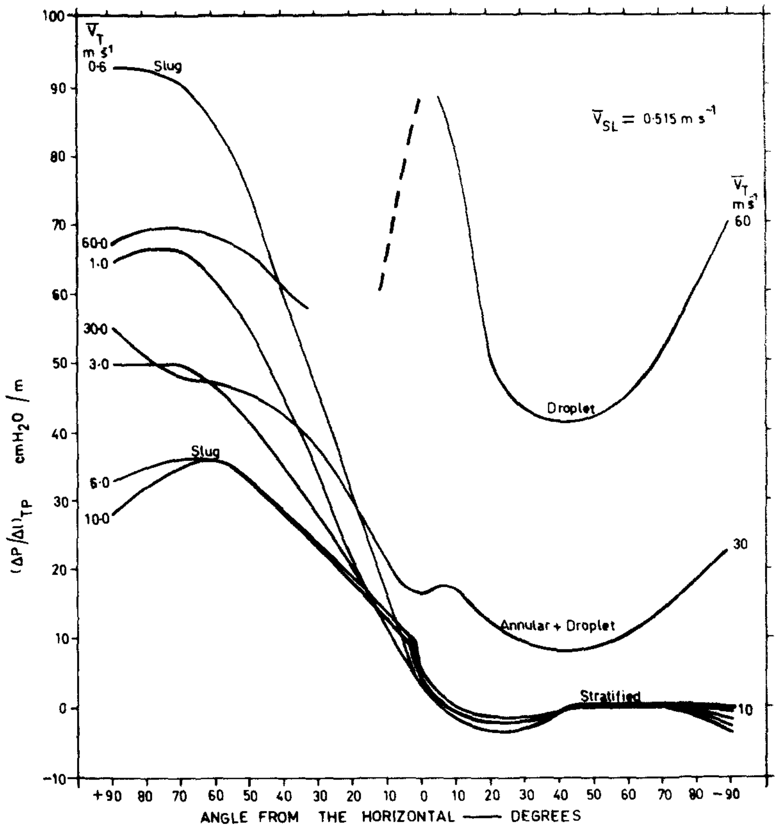

Figure 15. Two phase total pressure loss against angle of inclination for the air–water system in a 4.55 cm dia. tube at a superficial liquid velocity of 0.515 m s^{-1}. Data for the dashed portion of the $\bar{V}_T$ = 60 m s^{-1} curve were obtained by extrapolation.

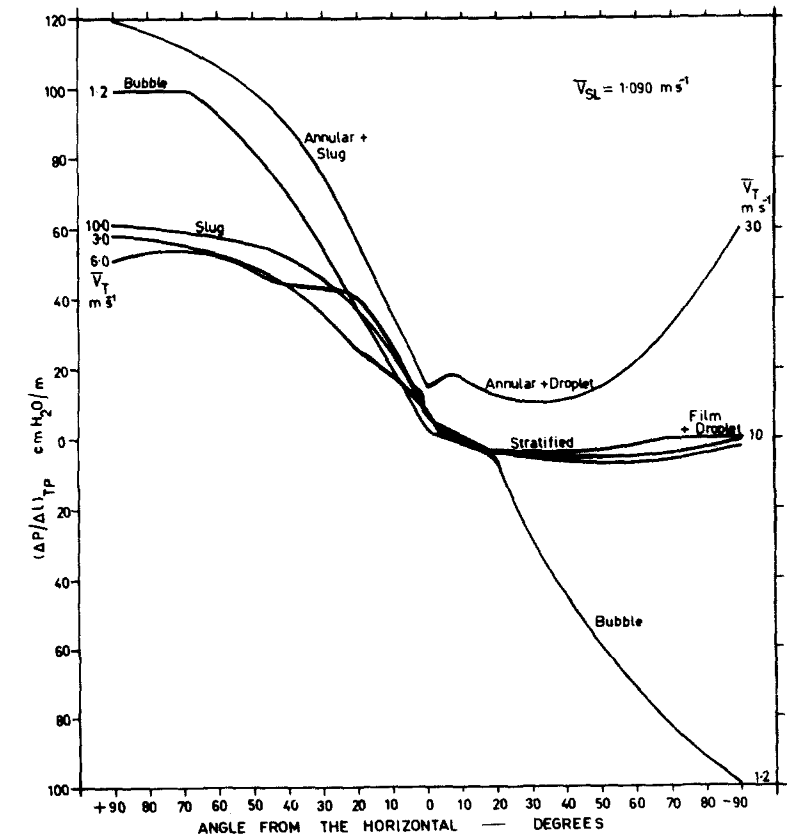

Figure 16. Two phase total pressure loss against angle of inclination for the air–water system in a 4.55 cm dia. tube at a superficial liquid velocity of 1.09 m s^{-1}

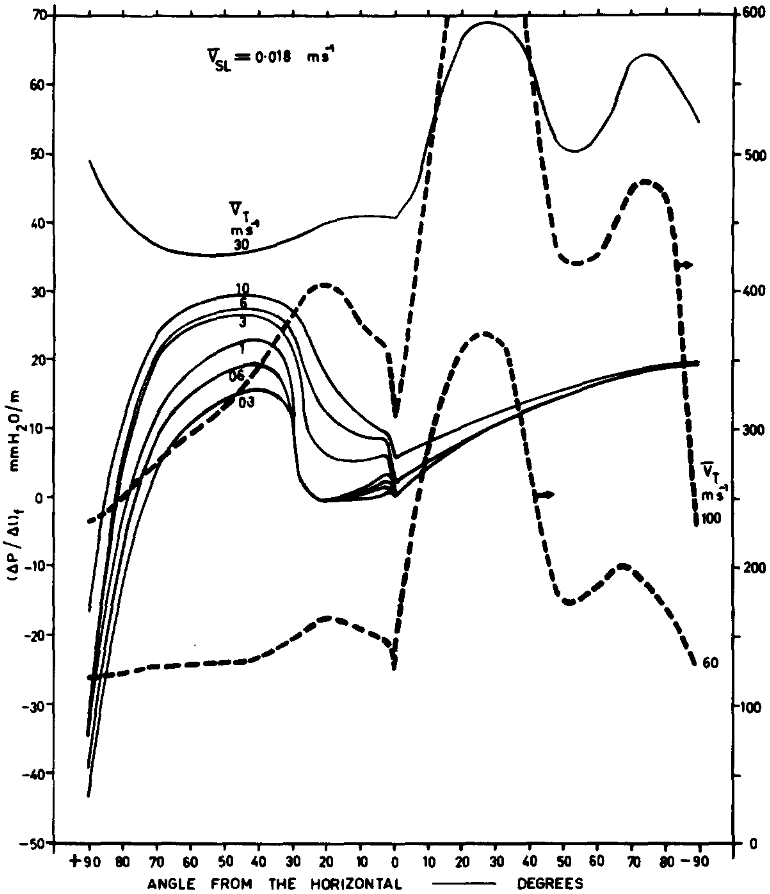

Figure 17. Two phase frictional pressure loss against angle of inclination for the air-water system in a 4.55 cm dia. tube at a superficial liquid velocity of 0.018 m s^{-1}. Two different frictional pressure loss scales are used for the full and dashed lines in the diagram.

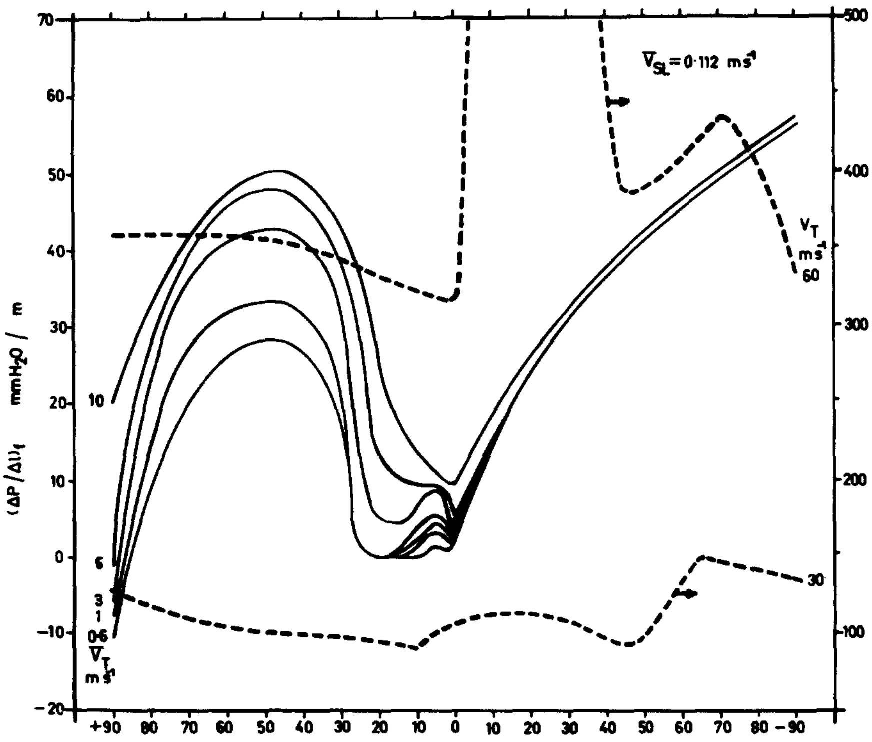

Figure 18. Two phase frictional pressure loss against angle of inclination for the air-water system in a 4.55 cm dia. tube at a superficial liquid velocity of 0.112 m s^{-1}. Two different frictional pressure loss scales are used for the full and dashed lines in the diagram.

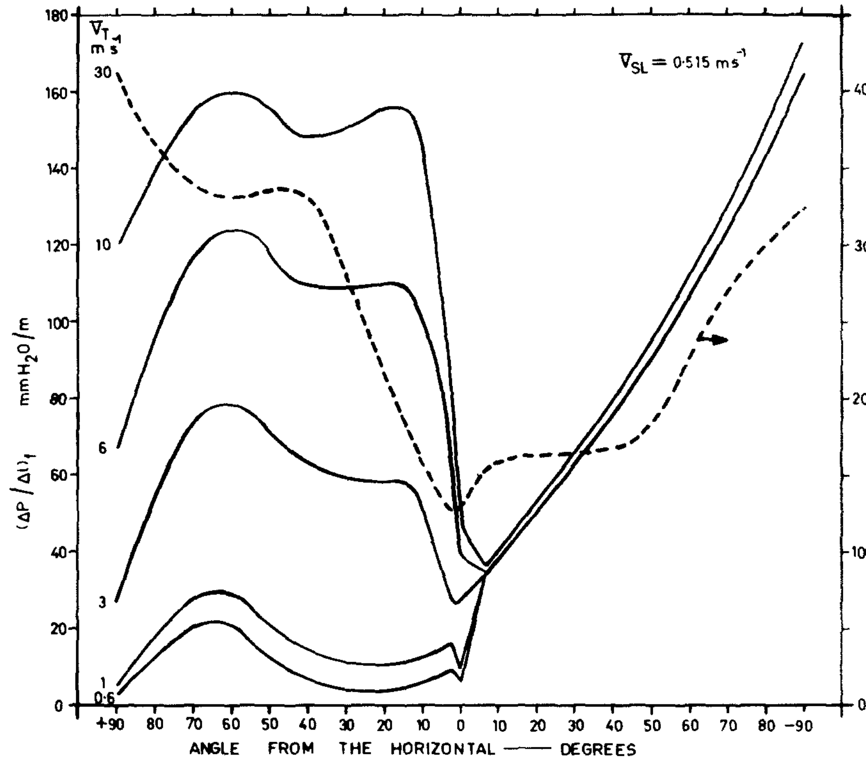

Figure 19. Two phase frictional pressure loss against angle of inclination for the air–water system in a 4.55 cm dia. tube at a superficial liquid velocity of 0.515 m s^{-1}. Two different frictional pressure loss scales are used for the full and dashed lines in the diagram.

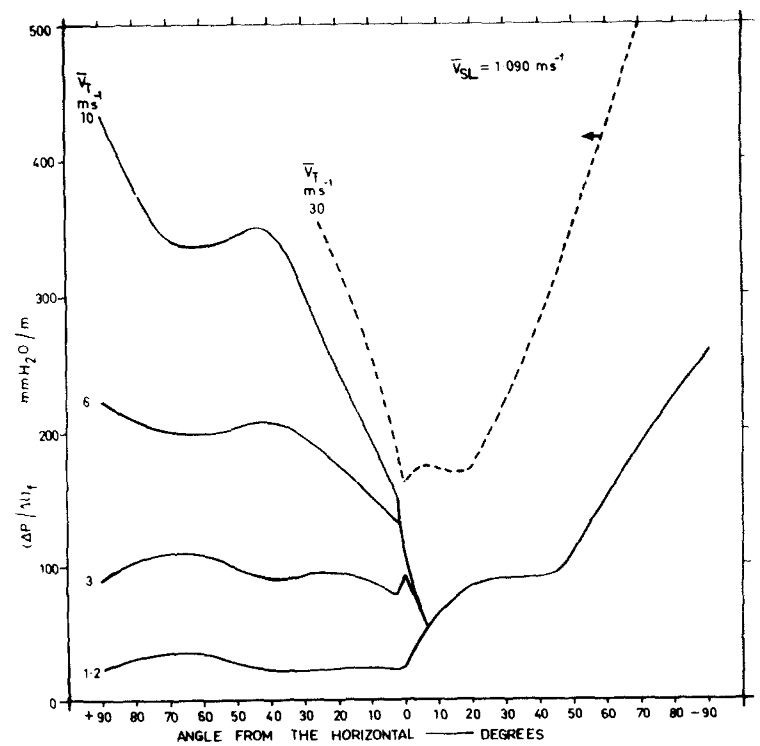

Figure 20. Two phase frictional pressure loss against angle of inclination for the air–water system in a 4.55 cm dia. tube at a superficial liquid velocity of 1.09 m s^{-1}. Some of the data for the dashed line was obtained by extrapolation.

downward flow there is a decided maximum in the total pressure loss at between −25° to −30° and a slight minimum at about −70°. In general as the total velocity is increased from $\bar{V}_T = 30\,\mathrm{ms}^{-1}$ the total pressure loss increases substantially, particularly the maximum at about −25° to −30°.

When the superficial liquid velocity is increased beyond $\bar{V}_{SL} = 0.4\,\mathrm{ms}^{-1}$ there is a progressive alteration in the total pressure loss characteristics. Again the relationship with angle of inclination can be separated into two distinct regions above and below $\bar{V}_T = 10$ to $30\,\mathrm{ms}^{-1}$ where a minimum in the total pressure loss is achieved. When $\bar{V}_T < 10\,\mathrm{ms}^{-1}$ a broad maximum is in evidence at about +90° to +70°, but as $\bar{V}_T$ is raised the maximum alters to about +60°. Thereafter the total pressure loss falls with decreasing angle to a negative minimum value at about −20°, rises to approximately zero over a broad range at angles between −40° to −70° and falls to a negative value of total pressure loss at −90°. At the high range of total velocity, $\bar{V}_T > 10$ to $30\,\mathrm{ms}^{-1}$, the total pressure loss at first falls steadily as the angle is decreased from +90° to give a minimum positive value at the horizontal condition. For downward flow a slight maximum is in evidence at about −10° and a minimum at about −45°. As the total flow rate is increased above $\bar{V}_T = 30\,\mathrm{ms}^{-1}$ the total pressure loss is increased substantially particularly at about −10° while a slight minimum appears at about +70°.

A detailed examination of the holdup data and flow regimes does give some explanation for the observed total pressure loss against angle of inclination characteristic. The region $\bar{V}_T = 10$ to $30\,\mathrm{ms}^{-1}$ is generally observed to be where the annular type flow regime appears except at very high superficial liquid velocities. When $\bar{V}_{SL} < 0.4\,\mathrm{ms}^{-1}$ the flow regime change in this region is from slug to annular plus droplet for upward flows while in downward and horizontal flows the regime change is from stratified to film plus droplet. As the superficial liquid velocity is increased beyond $\bar{V}_{SL} > 0.4\,\mathrm{ms}^{-1}$ at first the annular plus slug regime and finally the bubble regime dominates the whole of the upward flow so that no actual flow regime change is observable in the $\bar{V}_T = 10$ to $30\,\mathrm{ms}^{-1}$ region. However for downward flows there is first a change from stratified to annular plus roll wave when $\bar{V}_{SL} < 0.4\,\mathrm{ms}^{-1}$ in this region but as the superficial liquid velocity is increased the flow regime change observed is from film plus droplet to annular plus droplet. Thus it would appear that the observed change in the total pressure loss characteristic in the region $\bar{V}_T = 10$–$30\,\mathrm{ms}^{-1}$ is due in part ot a change in flow regime since the holdup $\bar{R}_G$ rises steadily with $\bar{V}_T$ in this region in a smooth and regular manner.

For vertical flow, the maximum in the total pressure loss that appeared at approximately 70° for low total velocities of $\bar{V}_T < 0.4\,\mathrm{ms}^{-1}$ is associated with a minimum in the gas holdup $\bar{R}_G$ for the slug regime. It is well known that disturbance of the region around the nose of the rising gas slug does cause hinderence to the liquid flow and it is conceivable that decreasing the angle of inclination from the vertical does cause such a distortion of the slug–liquid interfacial region to occur.

The other maximum in the total pressure loss characteristic was observed at about −20° on a broad peak. Here no observable change was noticed in the flow pattern while the gas holdup altered very little over the region of interest. Martin (1976) mentions the adverse effect of gas bouyance forces on two phase pressure loss in downward slug flow but it is improbable that such a mechanism could be involved in this case since liquid droplets and not gas bubbles are present, besides the effect is limited to a small range of all possible downward flow angles. Furthermore the effect is still in evidence in the frictional pressure loss data where corrections have been made for holdup. There needs to be further work done on this region of two phase flow, particularly visualisation experiments in order to attempt to assess what is physically happening in the region of the −20° angle.

Negative values of frictional pressure loss were obtained in the vertical flow situations with low liquid flow rates. Such negative frictional pressure losses have been observed by a number of workers, e.g. Nicklin (1962), and are due to the fact that the water film immediate to the wall is forced downwards against the flow by the rising gas slug. Naturally as the superficial liquid velocity is increased this effect will be eliminated as discussed by Spedding & Nguyen (1978). All the upward flow frictional pressure loss data given in Figures 17 and 20 exhibits a steady rise with increasing total velocity indicating that the corresponding minimum in the total pressure loss curve at about $\bar{V}_T \simeq 15\,ms^{-1}$ is due in no small part to a steady fall in liquid holdup culminating eventually in a change in flow regime in most cases. For the total velocity region $\bar{V}_T < 10\,ms^{-1}$ the frictional pressure loss for downward flow is virtually independent of total flow rate and is dependent only on superficial liquid velocity and angle of negative inclination. When the total flow velocity is increased above $\bar{V}_T = 10\,ms^{-1}$ a maximia in the frictional pressure loss begin to appear at between $-20°$ to $-30°$ and $-60°$ to $-70°$. The reason behind these maximia are obscure at this stage but the fact that they appear for both total and frictional pressure loss data shows that some fundamental reason is involved and not just simply a matter of variation in either liquid holdup or flow regime.

There are a number of operational and design criteria which are highlighted in this work which should be observed if energy losses are to be reduced in two phase systems. Firstly there will be a tendency to minimise the total pressure loss for vertical angle flow by operating with a total velocity of $\bar{V}_T \simeq 15\,ms^{-1}$. For horizontal and downward flows it appears to be better to operate at a total flow velocity of $\bar{V}_T \simeq 8\,ms^{-1}$. If total velocities substantially lower than $\bar{V}_T = 15\,ms^{-1}$ are used downward flow will cause little difficulty as far as a total pressure loss is concerned, but for upward flow the angle between $+60°$ and $+70°$ should be avoided. If total velocities are envisaged which are substantially higher than $\bar{V}_T = 15\,ms^{-1}$ upward flow will cause little difficulty since in this operation region total pressure loss is not very dependent on angle of inclination. However it is perhaps best to operate only in the vertical and the exactly horizontal mode. For downward flow the angle between $-45°$ and $-70°$ should be used while particularly the angle down to $-40°$ should be avoided. If the angle of pipe will be known to vary widely it is best to operate at a velocity of $\bar{V}_T = 15\,ms^{-1}$ since the pressure loss is a minimum for many operating angles and is substantially independent of inclination.

CONCLUSIONS

Two phase pressure loss data are presented for the air–water system for angle of inclination from vertical upward to vertical downward co-current flow. The data are presented as frictional velocity $\sqrt{((\Delta P/\Delta l)_f(D/4\rho_L))}$ against total velocity $\bar{V}_T$ and in this form successfully overcome the effect of pipe diameter, liquid density and system but not that of liquid viscosity. Detailed comparison with data from the literature provided a substantial measure of agreement. All of the empirical and semi-theoretical correlations failed to give any close agreement with the data of the present work. It appears, therefore, that it will be extremely difficult to device a general purpose prediction method for two-phase frictional pressure loss. This is because frictional pressure loss is so dependent on the flow regime present in the pipe.

The slug flow regime for certain cases in horizontal and inclined upward flow particularly should be avoided since it is characterised by high frictional pressure loss of a fluctuating nature. For vertical or near vertical flow, negative frictional pressure losses are obtained for the conditions of $\bar{V}_T < 18\,ms^{-1}$ and $\bar{V}_{SL} < 0.1\,ms^{-1}$. As a general rule the frictional velocity of downward flow is higher than the corresponding horizontal case but it is independent of total flow velocity for $V_T < 10\,ms^{-1}$. When the total flow velocity is substantially above this figure maximia occur in the frictional pressure loss at angles of inclination of $-20°$ to $-30°$ and $-60°$ to $-70°$.

Total pressure losses in this two phase system can be minimised by operating at a total velocity of $\bar{V}_T \simeq 15\,ms^{-1}$ for vertically upward angles of inclination. For horizontal and downward flows it is better to operate at a total velocity of $\bar{V}_T \simeq 8\,ms^{-1}$. When operating substantially below these figures of total velocity the vertical angle of $+60°$ to $+70°$ should be avoided and when operating substantially above these figures only the downward angle between $-45°$ and $-70°$ should be used.

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